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Task design for Engineering Mathematics: process, principles and products

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We present and analyse principles and process employed at the Danish Technical University to use authentic problems from engineering (APE) in a first year mathematics course, along with some of the products (actual student assignments).

Keywords: Mathematics for Engineers, Task Design

INTRODUCTION

Mathematics is as important in most branches of engineering, as engineering is to the prosperity and development of contemporary society. Thus, it is of great importance to investigate exactly *what* mathematics is needed by (future) engineers, and how it could be effectively taught to them; such research is only emerging (see e.g. Winsløw et al., to appear, sec. 2.5). In the common case where mathematics is taught in separate “service” courses which cater to several different study programmes, these questions may be considered in entire separation: a syllabus for the mathematics course is decided based on needs in the different study programmes of engineering which include the course, and subsequently the syllabus is delivered by mathematics faculty. This amounts to a complete separation of external and internal didactic transposition, in the sense of Chevallard (1992), where the selection of mathematical contents to be taught may be based on needs and priorities from the engineering disciplines, while the actual teaching is carried out according to generic standards and methods for teaching mathematics. The aims of the overall study programme (in engineering) are only considered in the external transposition of the mathematical knowledge (see Fig. 1). We can call this model a *parallel model* for teaching mathematics to engineers, as the internal didactic transposition runs in parallel to the rest of the programme and does not interact with it (while it is certainly intended that the students’ learning serves in other courses, later on).

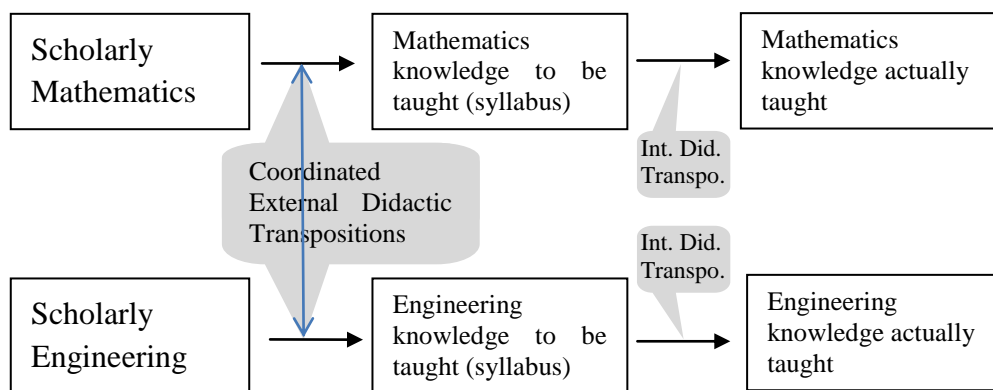


Figure 1: The parallel model for didactic transposition in engineering education

In the literature on university mathematics education, it is widely agreed that the parallel model has drawbacks:

- students may experience the mathematics teaching as unmotivated and difficult, which is reflected in relatively high failure and attrition rates for some engineering programs (e.g. Baillie & Fitzgerald, 2010).
- the knowledge they acquire in the mathematics course may not transfer readily to engineering contexts, in the sense that students are able to invest the knowledge acquired in mathematics courses when they need to do so in other courses of the programme (e.g. Britton et. al., 2005).

Motivated by these well-known problems, the model in Fig. 1 has been modified, in many universities, by various attempts to relate the internal didactic transposition of mathematics more closely to the rest of the engineering programs (e.g. Kumar & Jalkio, 1999).

One of the most common ideas to further such an interaction is that to include, in the mathematics course, more or less simple examples and student assignments where engineering problems are solved based on mathematical methods and theoretical notions (see, for instance, Härterich et al., 2012). A main challenge here is that university mathematics teachers usually have no in-depth knowledge of any engineering discipline, let alone of all the specialties which the course they teach caters to. Of course they may then ask engineering specialists for help to identify *authentic problems from Engineering which can be solved using the mathematics to be taught in their course* (we abbreviate this kind of problems as APE). In that way, “Scholarly Engineering” may exercise a more direct influence on the internal didactic transposition of mathematical knowledge (cf. Fig. 1). In this paper, we investigate some general questions related to the implementation of this (quite common) idea at the level of the internal didactic transposition:

RQ1. How could the identification and transposition of APEs be organised, given the academic and institutional separation of university mathematics teachers from their colleagues in engineering?

RQ2. What didactic variables (cf. eg. Gravesen, Grønbæk and Winsløw, 2017) are relevant to the construction of assignments based on APEs?

It is clear that answers to these questions will depend on institutional conditions and that even when such conditions are given, one will at most obtain very partial answers in the sense of reasonably validated *examples* of organisations (RQ1) and didactic variables (RQ2). As always in education, transfer of “answers” from one context to another will require some adaptation and interpretation, but this limitation may in fact be less important for the above questions, given the relatively high similarity of mathematics courses, the engineering programs they serve and the institution which deliver them. For these reasons, and given the importance of the matter already argued, it appears worthwhile to present such locally and partially validated answers. Concretely we will present and analyse the process, principles and

products of APE design done at the Technical University of Denmark since 2000. Founded by H. C. Ørsted in 1829, this is one of the most prestigious Schools of Engineering in Europe, and by far the largest in Denmark.

A TASK DESIGN PROCESS

Mathematics 1 (hereafter Mat1) is the basic mathematics course for 17 different B.Sc.Eng.-programmes at the university, catering to about 1100 students per year. The course occupies 1/3 of the students' time during the first year, and covers complex numbers, basics of Linear Algebra, Ordinary Differential Equations including linear systems, and multivariate and vector Calculus up to Gauss' theorem. Besides ensuring a technical foundation for later work, the university also considers a common course on mathematics as important to the formation of an engineer identity.

Most of the course is quite traditional, however with intense use of the computer algebra system *Maple*. Exercises with easy applications to engineering occur. However, during the last four weeks of the course, the students work on a "project". This is an assignment containing about 20-30 more or less challenging tasks, related to a mathematical model related to an APE. The model is usually given in the assignment, and while some new mathematics may be introduced, the starting point is Mat1. Each project assignment is presented in a text of varying extent (ranging from 4 to 29 pages, averaging 11); it is those texts which we aim to analyse in this paper. The students do the projects in groups, hand in a report of about 20-50 pages, and defend their work during an oral exam, which accounts for 25% of their grade.

The groups can choose their assignment from a list of 4-5 projects, in part depending on the study programme, with titles like those shown shown in Fig. 4. As the titles suggest, the project problems come from many different areas of Engineering. Every year, new projects are added and some are dropped; and the details of retained projects are updated based on teachers' experiences. The elaboration of new projects is a particularly delicate undertaking. When the first projects were done from 2000-2006, a systematic effort was deployed to engage researchers from the university – both applied mathematicians and researchers from Engineering at large – to propose project topics; they were then, mostly, drafted or adjusted by the course responsible. Some are still used in revised form.

It is the task of the course responsible to organise the production and revision of projects. The initiative can come from teachers at the course or other mathematicians, who identify a more or less classical APE which can form material for a project; then, the motivation is often that some specific parts of Mat1 can be worked on in new ways. But the initiative also frequently come from colleagues from other departments. In some cases, their motivation is personal fascination with mathematics in a more or less current APE, and possibly ongoing collaboration with mathematics colleagues in this relation (reflecting an interaction between Scholarly Mathematics and Engineering, which could be added to Fig. 1). In other cases, they propose an attractive current APE for a Mat1 project, in order to attract students to their specialty

later on – these colleagues then, sometimes participate for free as supervisors on the students' project work.

Summing up what the process involved in Mat1 could contribute to RQ1, at least two venues can be identified in relation to Fig. 1:

- Scholarly (applied) Mathematics and other basic sciences such as chemistry and physics, where the main source of motivation is mathematical contents related to Mat1; but work done here can still involve or lead to APE. Project proposals from this source are typically mathematically “rich” but are not often related to current research.
- Scholarly Engineering, often with current APE's; the elaboration of a project typically necessitates considerable adaption to fit Mat1, and is often tailored to the interest of students from a small range of study programmes.

Finally, the genesis of a project may involve a mixture of both sources, when the APE is identified by scholars with a deep involvement in both areas.

DIDACTIC VARIABLES AND PRINCIPLES

To present and analyse the project assignments which have appeared over the last 15 years, we have defined 10 *didactic variables* (DV) which are relevant to classify them according to the aims which have, explicitly or implicitly, been pursued (Fig. 2). Each variable has, in principle, a non-numerical range, but can be determined with relatively high objectivity for each assignment, based on the text. The detailed presentation of any project in terms of the variables will, naturally, be difficult to compare with others when given in this form. So when considering all projects it appears useful to assign indicatory numerical values to the DVs on a scale from 0 to 2: for instance, to assess the breadth of Mat1 contents which a given project requires the students to work with, 0 indicates that only one topic (such as systems of linear equations) is involved, 1 that a few topics from both Calculus and Linear Algebra are involved, and 2 that the project combines many topics. Naturally, this “grading” is not absolute but relative to other projects (cf. also Fig.2). In the next section, we outline a concrete assignment and explain, at the same time, how the numerical values of the other DV's are set. The variables were initially formulated in by the authors (based on the first authors' many years of involvement in the design) and subsequently validated and adapted during the actual analysis of assignments. The variables thus constitute a concrete answer to RQ2, which is of course a partial answer based on experiences from context we described. In the rest of this paper, we provide more explanation on how the variables can be used to analyse concrete projects and, potentially, to direct and systematize the design of student assignments.

For each DV, Fig. 2 also includes a brief description of the more or less explicit aims which have been pursued in the construction of projects over the past 17 years. The brevity required in the Table format does not allow for much nuance. We note that what is ideal use of *Maple* is not subject to complete consensus among the teachers of the course, or in relation to the rest of the institution. On the one hand, some course

teachers consider that students should use *Maple* whenever it is useful; while others, including colleagues from other departments, often insist on the value of students' mastery of basic manual computation (as reflected in the aims for DV3 given in Fig. 2). The variables DV4-10 all describe aspects of the relationship between the internal transposition represented by the assignment, and Scholarly Engineering (cf. Fig. 1). Their values are thus of specific importance to go beyond the parallel transposition.

Didactic variable (DV):	Aim of designers:
DV1 What breadth of content areas from Mat1 are needed to solve the assignment? What depth of use?	As many as possible, preferably involving new combinations. Depth beyond "standard tasks" required.
DV2 What new mathematical contents are introduced?	Contents in continuation of Mat1, not excessive for students to cope with
DV3 How must/can <i>Maple</i> be used?	<i>Maple</i> should mostly be used to:
<ul style="list-style-type: none"> - DV3a How <i>essential</i> is the <i>Maple</i> use? - DV3b What types of <i>Maple</i> functions (numerical, symbolic, graphical...) are relevant? - DV3c Are the relevant use known or new to students? - DV3d Is there <i>black box</i> use of <i>Maple</i> ? - DV3e What parts of the <i>Maple</i> use are prescribed? 	avoid tedious computations, and for tasks which the students could not handle otherwise
DV4. What is the "theme" and source of the problem the project attacks	Origin in APE, if possible source in paper or ongoing research in engineering
DV5. Breadth of engineering problem – are more disciplines involved?	Ideally more than one branch of engineering involved
DV6. How is the mathematical model established and worked with?	Ok if model is given in the assignment, but the students should work with its details and structure
DV7. How realistic is the model?	As much as possible for the students
DV8. How are data used?	Data from the source, used as there
DV9. Should the students look up information outside assignment?	This is not a main aim, except students should use Mat1 course material
DV10. How complete answers does the model give to the main problem?	Clear and definite answers/points, to give students a satisfying experience

Figure 2. Didactic variables for the analysis of project assignments.

PRODUCTS

A total of 37 projects have been proposed during the past 10 years. Not all projects are used every year, and all are revised before use, in the light of past experience, new needs in the course, and in a few cases, updates to the APE and its solution from

Scholarly Engineering. We first give a relatively detailed presentation and analysis (based on the DVs) of one project; at the same time, we describe how each of the 10 DVs is assigned a value as described above, for a rough analysis of the projects. Then we present an overview and rough analysis of the whole inventory of projects.

In-depth presentation of one project

We now take a closer look at one of the projects, entitled: *Heat flow in a house – simulation and dimensioning*. The assignment is relatively long, 18 pages, including about 5 pages of data. The first paragraph outlines the underlying APE:

The building sector accounts for about 40% of the total energy consumption in Denmark. It is a common assumption that there is a large unrealized potential for reducing the consumption (...) in a financially sound way. This requires knowledge of the physical processes which affect the energy consumption of buildings, the financial aspects of the construction and maintenance of buildings, as well as the mathematical methods used to compute these.

It turns out that the energy flow in the building is modelled as an analogy of currents in electric circuits (cf. Fig. 3). The project is based on a genuine APE, and bibliography of the assignment includes a reference to Nielsen (2005) which is the essential source (DV4 = 2), along with a “pricelist” from the construction industry, and the last part of the assignment draws on a simple model of investments and interest. Relative to other projects, this assignment involves a relatively broad area of Engineering fields, and DV5 is set to 2. The introduction acknowledges that the model proposed in assignment is “a bit simplified”, but in fact it still gives similar results; we assign DV7 to 2, in spite of some problems (see end of this section).

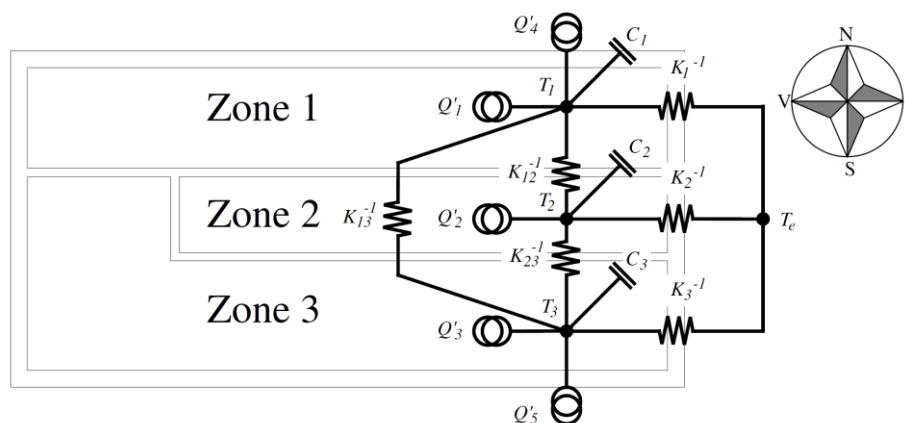


Figure 3. A figure from the project assignment “Heat flow in house”.

The central model, illustrated in Fig. 3, concerns a house with three rooms, called “climate zones”. Here Q'_k are the internal and external heat sources (heaters and sunlight), while C_k are the heat capacities of the rooms and K_* are the heat transmission coefficient of the walls of the house, reflecting that these walls involve a variety of layers. Before introducing the final model house, the students’ work with the simpler case of a one-room house, and an external temperature T_e which is a

given sine function. Based on further assumptions, this leads to the model for the internal temperature T_i , as a function of time t :

$$(*) \quad C \frac{dT_i}{dt} = K(T_e - T_i) + P(T_{set} - T_i)$$

where T_{set} is the desired internal temperature (constant), and P is the performance of the internal heat source. (*) is a first order ODE, but still gives rise to interesting Engineering tasks: the investigation of stationary solutions, the performance needed to ensure an average temperature of 19.8°C , and the thickness required to respect given limits on the oscillation of T_i . The full model consists of a system of three differential equations which are similar to (*) but with an added complexity due to the heat contribution from sunlight which, moreover, is investigated with two different models. The students must also take into account a model of the walls involving layers of materials to be computed using authentic data. Finally, to take into account the cost of construction, the students are given a simple mathematical model for the total economy involving investment, interest, and operation costs; the mathematics is very simple but still gives rise to interesting questions regarding how to optimize, for instance, insulation thickness. Throughout, the students use real data (DV8=2) but these are all given, so DV9=0. Throughout the models are given to the students, and while students are given full and extensive explanations, they are not really asked to do more than apply them; thus we set DV6 at 1.

The project draws on a broad range of Mat1-topics: harmonic oscillations and complex exponential function, single and coupled differential equations, solved using advanced matrix algebra, involving both eigenvalue problems and quadratic forms. Thus DV1=2, while DV2=0 as almost no new mathematics is introduced (the exception being the argument required to justify the stationary solution to the system of differential equations, which involves an extended eigenvalue problem).

While most of the tasks can in principle be solved manually, the visualizations of temperature variations corresponding to different parameter values decisively require a tool like *Maple*. The assignment moreover invites to numeric experimentations, possibly based on graphical representations, and standard use for tedious operations like inversion of matrices, make the overall potential of *Maple*-use relatively average for projects; we thus assign DV3=1, even if the realized use by some students in some cases goes beyond a mere use of techniques known from the rest of the course.

In real practice, the project also suffers from some flaws. Some of the questions lead to less interesting results (like tedious computations leading to a requirement of a four-doubling of the wall thickness in order to reduce an already negligible oscillation of 0.12°C for T_i). More serious is the breakdown of the model when taking into account the contribution of sunshine at low temperatures such as 8°C , where the stationary answer cannot be found. It can be argued that such problems often arise with simplified models, but it still leaves a negative impression on many students, which might be avoided by future revisions. Altogether, we consider DV10=1.

Inventory of project assignments

Fig. 4 lists the inventory of projects used in the last 10 years, analysed using the DVs.

	DV (cf. Fig. 2)									
Project name (shortened in a few cases)	1	2	3	4	5	6	7	8	9	10
Oscillations in Axle-bearing Systems	2	0	1	2	1	1	2	2	1	2
Micro/Nano Cantilever Based Mass Sensor	1	2	1	2	1	1	2	1	1	1
Enzymatic Hydrolysis of Cellulose	1	1	1	0	1	2	1	1	0	1
Modelling 2D Halbach permanent Magnets	1	2	2	2	1	2	2	1	0	2
Factorization of Integers	0	2	2	0	1	0	1	0	0	1
Heat flow in house – simulation, dimensioning	2	1	2	2	2	1	2	2	0	1
Quantum Mechanics in a Nutshell	2	0	1	2	2	1	2	2	0	1
Red Blood Cells – Optimization in Nature	1	2	2	1	1	2	1	0	0	2
Utilization of the Waste Product Whey	1	0	1	2	1	2	1	1	0	2
Forced Pendulum	1	2	2	1	1	2	1	0	0	2
Stability in Chilled Tank Reactor	1	2	1	1	1	1	1	1	0	2
Optimization of Work Cycles	2	1	2	2	1	1	1	0	0	2
GPS and Geometry	1	2	2	2	2	1	1	1	2	1
Oscillations in Grid Constructions	2	2	1	2	1	2	1	0	1	1
Groundwater Flow in the Forest Vestskoven	1	2	2	2	1	1	2	2	0	1
Internet Hit lists	1	2	2	2	1	1	2	2	1	2
Short Circuit in Electric Networks	1	2	2	1	1	1	1	0	0	2
Simulation of Stretch Reflex	1	1	1	2	1	1	1	0	0	1
Parking Orbits of Satellites	2	1	0	1	1	2	1	0	1	2
Solar Energy Absorption in Curved Glass houses	2	0	2	1	1	1	1	0	0	1
Flow in Chemical Reactors	2	0	1	1	1	2	1	0	1	2
Finite elements in One Dimension	1	2	2	2	1	2	1	0	1	1
Geodesic Curves	1	2	2	0	0	2	2	0	0	2
The Brains Glycose Metabolism	1	2	2	2	1	1	2	1	1	1
Resistors and Markov Chains	1	2	2	1	1	1	1	0	0	2
Dosage of Anaesthesia	2	1	1	2	1	2	1	1	0	2
Anthrax – Attack, Escape and Rescue	1	2	2	2	1	2	1	1	2	2

Decomposition of PCE	1	1	1	2	1	1	1	2	1	0
Modelling Concrete Moulding	1	2	2	2	2	1	2	2	2	1
Soap Membranes	1	2	2	1	1	1	2	0	0	1
Distribution of Electrons in Semiconductors	1	2	2	1	1	1	2	0	0	1
Methane Concentration Profiles in Soil	1	2	1	1	1	1	1	1	0	2
Train Running in the Alps	2	1	2	1	2	2	1	0	1	1
Proteins' 3 Dimensional Structure	0	2	2	2	1	1	1	2	1	1
Reaction Kinetics	1	2	2	1	1	1	2	1	0	2
Error Correcting Codes	0	2	2	1	1	1	1	0	0	2
Phononic bandgaps	2	0	2	2	2	2	1	0	0	2

Figure 4: Inventory of current projects with values of the didactic variables

A number of interesting tendencies can be identified in the above table, including apparent dependencies of some variables, potentials which appear relatively unexplored (like DV9), etc.; some of these are still not fully analysed. We stress that a simple sum of the values of didactic variables, for a specific project, cannot be construed as a measure of the “didactic quality” of the assignment. One reason is that the variables are not of equal importance (in particular, DV1, 4 and 10 are essential). But more importantly, one cannot always construe the number two as being objectively “the best possible value” of the DV; the aims listed in Figure 2 are open to debate and the viewpoint of teachers and designers may differ. A good example is DV4, where we have given “2” for projects with a clear APE, “1” for projects with an authentic problem from basic science (e.g. Chemistry) and “0” for projects which are not based on an APE but on a (*prima facie*) purely mathematical problem, such as the project “Geodesic curves”. One can argue that a project of the “0” type can also be of high quality as a project for engineers, in view of the importance in several branches of the mathematical problem (in the example, DV2=2 and indeed, geodesic curves have multiple applications in many branches of engineering, see e.g. Patrikalakis and Maekawa, 2010, 265-291). A similar uncertainty must also be pointed out for other variables such as DV3, where the further graduation suggested in Fig.2 could be useful to provide a more nuanced picture than in the analysis in Fig. 4, where “2” merely means that *Maple* is indispensable for large parts of the project.

SUMMARY AND OUTLOOK

We have presented the principles, process and products of a relatively longstanding effort to integrate elements of scholarly Engineering (APE's) in the internal didactic transposition of basic mathematics in a course catering to a wide range of Engineering programmes, going well beyond isolated “applications” of a Calculus or Linear Algebra. We have emphasised the multiple dimensions which such an effort

needs to consider, in order to maintain the link with the mathematical knowledge to be taught within the module in question, and to establish non-trivial links with Scholarly Engineering (cf. Figure 1). Certainly, the concrete inventory of variables can be developed and adapted further, and we believe it can eventually become a valuable explicit basis for the discussion of aims (right column in Fig. 3) of projects in our and other similar contexts. More importantly, considering such explicit variables could be an important tool for systematizing the design process, both as a check list for constructing new projects and (in combination with the analyses behind Figure 4) to identify potentials for enriching existing projects. We expect that the variables will also become useful guidelines for investigating the effects of the project work in this course as a means to facilitate the transition to later courses where mathematics is so fully integrated into the Engineering knowledge to be taught that the latter is in practice as inseparable from mathematics as music is from sound.

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